High-Order and Bias-Free Distributed Langevin Dynamics for Scalable Bayesian Inference

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Jan 30, 2026

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Background and Preliminaries

The overdamped Langevin diffusion is defined by the stochastic differential equation (SDE)[1] [2]:

$$dX_t = -\nabla f(X_t)dt + \sqrt{2}dW_t, \qquad t \geq 0 \tag{1}$$

where $\{X_t : t \in \mathbb{R}_+\}$ is a continuous-time diffusion process, $W_t : t \geq 0$ is a d-dimensional Brownian motion

- Under some mild conditions, the SDE in (1) has a strong solution that admits Gibbs distribution $\pi(x) \propto e^{-f(x)}$ as the unique invariant distribution.

[1] Pavliotis, G.A. (2014). Stochastic Processes and Applications: Diffusion Processes, The Fokker-Planck and Langevin Equations. Springer.

[2] Roberts, G.O. and Tweedie, R.L. (1996). Exponential convergence of Langevin distributions and their discrete approximations. Bernoulli. 2(4):341-363.

Background and Preliminaries

- The (first-order) overdamped Langevin Monte Carlo (LMC) algorithm is a well-known sampling method that is the Euler-Maruyama discretization of the SDE (1):

$$x_{k+1} = x_k - \eta \nabla f(x_k) + \sqrt{2\eta}\xi_{k+1} \tag{2}$$

- A variant of (1) is the (second-order) underdamped LMC algorithm (uLMC), that is based on the SDE:

$$\begin{aligned} dV_t &= -\gamma V_t dt - \nabla f(X_t)dt + \sqrt{2\gamma}dW_t, \qquad t \geq 0, \\ dX_t &= V_t dt, \end{aligned} \tag{3}$$

where $(V_t, X_t) \in \mathbb{R}^d \times \mathbb{R}^d$, $W_t : t \geq 0$ is a d-dimensional Brownian motion, $\gamma > 0$ is the friction coefficient

Background and Preliminaries

- Many novel Langevin algorithms have been proposed based on the optimization literature.
- Decentralized Stochastic Gradient Langevin Dynamics[3]

$$x_i^{(k+1)} = \sum_{j \in \Omega_i} W_{ij} x_j^{(k)} - \eta \tilde{\nabla} f_i \left(x_i^{(k)} \right) + \sqrt{2\eta} \xi_i^{(k+1)}. \tag{4}$$

  where W_{ij} are the entries of a doubly stochastic weight matrix W with $W_{ij} > 0$ only if i is connected to j.
- Projected Langevin Monte Carlo[4]

$$dX_t = \mathcal{P}_C \left(x_k - \eta \nabla f \left(x_k \right) + \sqrt{2\eta} \xi_{k+1} \right) \tag{5}$$

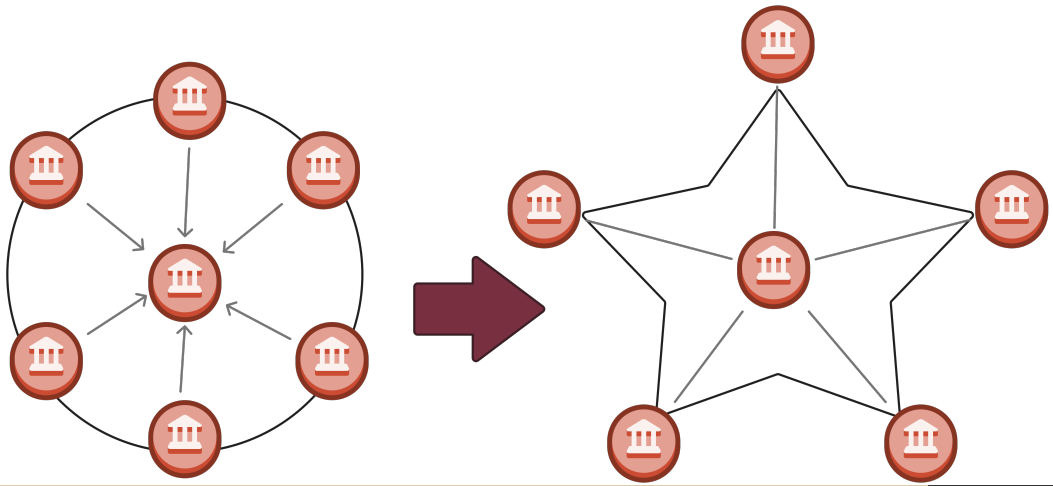
  where $\mathcal{P}_C(\cdot)$ is the projection onto the closed-convex set C.

[3] Gurbuzbalaban, M., Gao, X., Hu, Y. and L. Zhu (2021). Decentralized stochastic gradient Langevin dynamics and Hamiltonian Monte Carlo. Journal of Machine Learning Research. 22, 1-69.

[4] Bubeck et al. (2015). Finite-time analysis of Projected Langevin Monte Carlo. Advances in Neural Information Processing Systems.

Decentralized Learning System

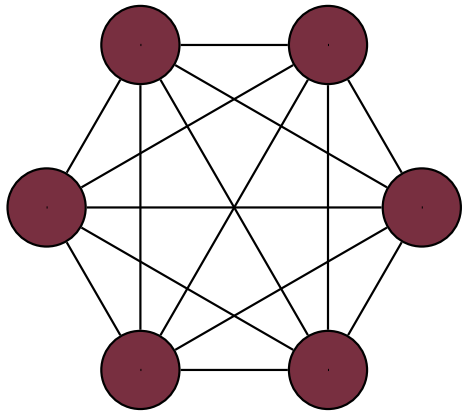
- The present era is the era of Machine Learning (ML), Artificial Intelligence (AI), and Big Data Analytics.
- The rate at which the data is generated is often outpaced by our ability to analyze in terms of computational resources. This causes the need of scalable machine learning algorithms.
- The data are collected through many digital devices, and those devices are connected through a communication network



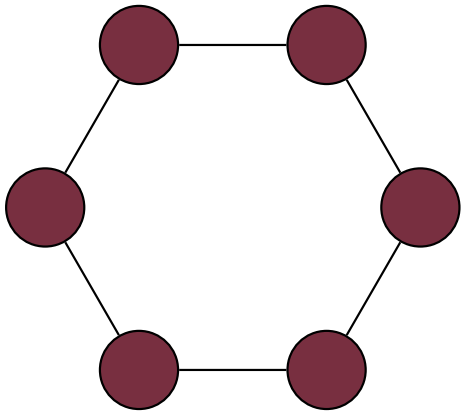
Decentralized Learning System

- We have N computational agents or nodes connected over a network $\mathcal{G}$ where $\mathcal{V} = \{1, 2, \cdots, N\}$ represents the agents and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges; i.e., i and j are connected if $(i, j) \in \mathcal{E}$.
- Let $A = [a_1, a_2, \cdots, a_n]$ be the input-output data pair with n i.i.d. observations sampled from a parametersized distribution $p(A \mid x)$ where the parameter $x \in \mathbb{R}^d$ has a common prior distribution $p(x)$
- In the decentralized setting, agent i possesses a subset A_i of the data where $A_i = \{a_1^i, a_2^i, \cdots, a_{n_i}^i\}$, and n_i is the number of samples of the agent i.
- The data is held disjointly; i.e., $A = \bigcup_i A_i$ with $A_i \cap A_j = \emptyset$ for $j \neq i$.
- The goal is to sample from the posterior distribution $p(x \mid A) \propto p(A \mid x)p(x)$

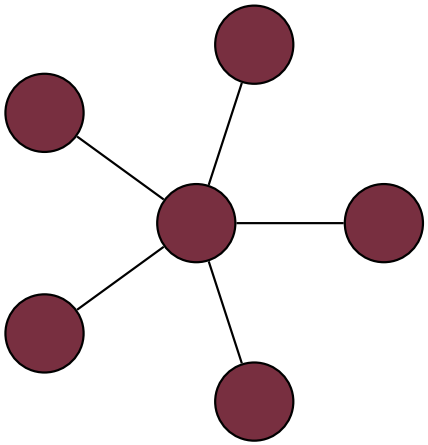
Network Topologies



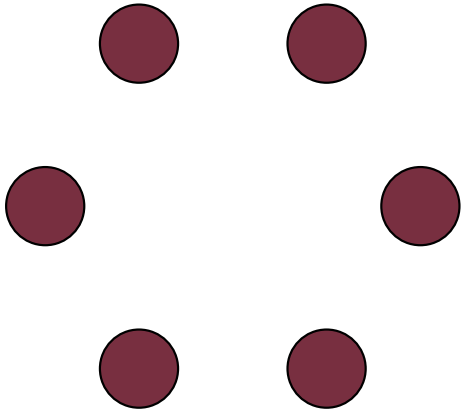
(a) Fully connected



(b) Circular



(c) Star



(d) Disconnected

Figure 2: Different types of network structures.

Decentralized Learning System

- Since the data are i.i.d., the log-likelihood is additive, i.e.,
$$\log p(A \mid x) = \sum_{i=1}^{N} \sum_{j=1}^{n_i} \log p(a_j^i \mid x)$$
- Thus, if we set the potential
$$f(x) := \sum_{i=1}^{N} f_i(x), \quad f_i(x) := -\sum_{j=1}^{n_i} \log p\left(a_j^i \mid x\right) - \frac{1}{N} \log p(x) \tag{6}$$
then the goal is to sample from the posterior distribution with density $\pi(x) := p(x \mid A) \propto e^{-f(x)}$
- The functions $f_i(x)$ are called "component functions" where $f_i(x)$ is associated tot he local data of agent i and is only accessible by the agent i.

Decentralized SGLD (DE-SGLD)

- Let $x_i^{(k)}$ denote the local variable of node i at iteration k.
- The decentralized SGLD (DE-SGLD) algorithm consists of a weighted averaging with the local variables $x_j^{(k)}$ of node i's immediate neighbors $j \in \Omega_i := \{j : (i,j) \in \mathcal{G}\}$ as well as stochastic gradient step over the node's component function $f_i(x)$:

$$x_i^{(k+1)} = \sum_{j \in \Omega_i} W_{ij} x_j^{(k)} - \eta \tilde{\nabla} f_i\left(x_i^{(k)}\right) + \sqrt{2\eta} w_i^{(k+1)}. \qquad (7)$$

  - $\eta > 0$ is the stepsize.
  - W_{ij} are the entries of the doubly stochastic weight matrix W. For instance, $W = I - \delta L$ where $\delta > 0$ and $L = D - A$ is the graph Laplacian.

Decentralized SGLD (DE-SGLD)

- $w_i^{(k+1)}$ are the i.i.d. and $w_i^{(k+1)} \sim \mathcal{N}(0, \mathrm{I})$
- $\tilde{\nabla} f_i\left(x_i^{(k)}\right)$ is an unbiased stochastic estimate of the deterministic gradient $\nabla f_i\left(x_i^{(k)}\right)$ with bounded variance, i.e., $\xi_i^{(k+1)} := \tilde{\nabla} f_i\left(x_i^{(k)}\right) - \nabla f_i\left(x_i^{(k)}\right)$ and

$$\mathbb{E}\left[\xi_i^{(k+1)} \mid \mathcal{F}_k\right] = 0, \quad \mathbb{E}\left\|\xi_i^{(k+1)}\right\|^2 \leq \sigma^2$$

where $\mathcal{F}_k$ is the natural filtration of the iterates $x_i^{(k)}$ up to (and including) k.

Why and What is EXTRA SGLD?

- Notice that when we use the full-batch, the iterations in (7) turn out to be

$$x_i^{(k+1)} = \sum_{j \in \Omega_i} W_{ij} x_j^{(k)} - \eta \nabla f_i \left(x_i^{(k)} \right) - \eta \xi_i^{(k+1)} + \sqrt{2\eta} w_i^{(k+1)}. \quad (8)$$

- The full-batch implementation introduces a bias term in the form of the Gaussian noise $\xi_i^{(k+1)}$
- As a result, the distribution of the iterates $x_i^{(k)}$ converges linearly to the neighborhood but not too close to the target distribution.
- Thus, we introduce EXact FirsT-OrdeR Algorithm (EXTRA) SGLD that allows to obtain non-asymptotic performance guarantees in terms of $\mathcal{W}_2$ distance.
- When each $f_i(x)$ and $f(x) = \sum f_i(x)$ are smooth and strongly convex, EXTRA iterates distribution converges to $\pi(x)$ linearly (but geometrically fast in k)

Wasserstein Distance

Define $\mathrm{P}(\mathbb{R}^d)$ as the space consisting of all the Borel probability measures μ on $\mathbb{R}^d$ with the finite second moment (based on the Euclidean norm).

Definition (2–Wasserstein distance)

For any two Borel probability measures μ, ν on $\mathbb{R}^d$ with finite second moments, the 2–Wasserstein distance[5] is defined as

$$\begin{aligned} \mathcal{W}_2(\mu,\nu) &:= \left(\inf \mathbb{E}\,\|x-y\|^2\right)^{\frac{1}{2}} \\ &= \left(\inf_{\gamma\in\Pi(\mu,\nu)} \int_{\mathbb{R}^d\times\mathbb{R}^d} \|x-y\|^2 \, d\gamma(x,y)\right)^{\frac{1}{2}}, \end{aligned}$$

where the infimum is taken over all joint distributions of the random variable x, y with marginal distributions μ, ν respectively.

[5] Villani, Cédric. Optimal transport: old and new. Vol. 338. Berlin: springer, 2008.

Assumptions

Assumption 1

We assume for every $i = 1, \cdots, N$:

- $f_i(x)$ is $\mu-$strongly convex and L-smooth, i.e., for every $x, y \in \mathbb{R}^d$,

$$\frac{L}{2}\|x-y\|^2 \geq f_i(x) - f_i(y) - \nabla f_i(y)^\top (x-y) \geq \frac{\mu}{2}\|x-y\|^2 \quad (9)$$

- $F : \mathbb{R}^{Nd} \mapsto \mathbb{R}$ with $F(x_1, \cdots, x_N) = \sum_{i=1}^N f_i(x_i)$ is also μ-strongly convex and L-smooth for any $x = (x_1, x_2, \cdots, x_N) \in \mathbb{R}^{Nd}$.

Assumptions

Assumption 2

The gradient noise $\xi_i^{(k+1)} := \tilde{\nabla} f_i\left(x_i^{(k)}\right) - \nabla f\left(x_i^{(k)}\right)$ at iteration k for any agent i is unbiased with a finite second moment, i.e.,

$$\mathbb{E}\left[\xi_i^{(k+1)} \mid \mathcal{F}_k\right] = 0, \qquad \mathbb{E}\left\|\xi_i^{(k+1)}\right\|^2 \leq \sigma^2, \tag{10}$$

where $\mathcal{F}_k$ is the natural filtration of the iterates $x_i^{(k)}$ up to (and including) time k.

Assumptions

Assumption 3

Consider a connected network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consisting of a set of agents $\mathcal{V} = \{1, 2, \cdots, N\}$ and a set of undirected edges $\mathcal{E}$. The doubly stochastic matrices $W = [W_{ij}] \in \mathbb{R}^{N\times N}$ and $= [\tilde{W}_{ij}] \in \mathbb{R}^{N\times N}$ satisfy

1. Null space property:

$$\text{null}\{W - \tilde{W}\} = \text{span}\{1_N\}, \qquad \text{null}\{I_N - \tilde{W}\} \supseteq \text{span}\{1_N\},$$

   where $\text{span}\{1_N\}$ is the span of the vector space supported by all-one vector $[1_N^\top, 1_N^\top, \cdots, 1_N^\top]$.

2. Spectral property:

$$\tilde{W} \succ 0, \qquad \frac{I_N + W}{2} \succeq \tilde{W} \succeq W.$$

Generalized EXTRA SGLD

- We assume that $\tilde{W} = h\, I_N + (1-h)W$, for $h \in (0, 1/2]$.
- Then the EXTRA stochastic gradient Langevin dynamics[6] as follows.

$$x_i^{(k+2)} = \sum_{j \in \Omega_i} W_{ij} x_j^{(k+1)} - \eta \nabla f_i \left(x_i^{(k+1)} \right) - \eta \xi_i^{(k+1)} + \sqrt{2\eta} w_i^{(k+2)}, \tag{11}$$

$$x_i^{(k+1)} = \sum_{j \in \Omega_i} \widetilde{W}_{ij} x_j^{(k)} - \eta \nabla f_i \left(x_i^{(k)} \right) - \eta \xi_i^{(k)} + \sqrt{2\eta} w_i^{(k+1)}. \tag{12}$$

[6] Gurbuzbalaban, Mert, Mohammad Rafiqul Islam, Xiaoyu Wang, and Lingjiong Zhu. "Generalized EXTRA stochastic gradient Langevin dynamics." arXiv preprint arXiv:2412.01993 (2024).

Generalized EXTRA SGLD

- These updates for N agents can also be expressed as

$$x^{(k+2)} = \mathcal{W}x^{(k+1)} - \eta\nabla F\left(x^{(k+1)}\right) - \eta\xi^{(k+1)} + \sqrt{2\eta}w^{(k+2)}, \tag{13}$$

$$x^{(k+1)} = \widetilde{\mathcal{W}}x^{(k)} - \eta\nabla F\left(x^{(k)}\right) - \eta\xi^{(k)} + \sqrt{2\eta}w^{(k+1)}, \tag{14}$$

- where $\mathcal{W} = W \otimes I_d$, $\widetilde{\mathcal{W}} = \widetilde{W} \otimes I_d$,
- $x^{(k)} = \left[\left(x_1^{(k)}\right)^T, \ldots, \left(x_N^{(k)}\right)^T\right]^T \in \mathbb{R}^{Nd}$, and
- $w^{(k)} = \left[\left(w_1^{(k)}\right)^T, \ldots, \left(w_N^{(k)}\right)^T\right]^T, \quad k = 0, 1, 2, \ldots,$

Generalized EXTRA SGLD

Let us define the average at the k-th iteration $\bar{x}^{(k)} := \frac{1}{N}\sum_{i=1}^{N} x_i^{(k)}$.

Theorem

Consider the generalized EXTRA Langevin dynamics with the network averaging matrix $\widetilde{W} = hI_N + (1-h)W$ where

$$0 < h \leq \frac{1-\bar{\gamma}_W}{4\bar{\gamma}^2_{I_N-W}} \wedge \frac{1}{2} \wedge \frac{1}{\gamma_1\gamma_2}, \tag{15}$$

and assume that the stepsize η is chosen satisfying

$$0 < \eta < \frac{1}{h\gamma_1\gamma_2} \wedge \frac{\gamma_{\tilde{W}}}{6(L+\mu)\vee 2A} \wedge 1 \wedge \frac{1}{L+\mu} \wedge \frac{\gamma_{\tilde{W}}}{6(L+\mu)}, \tag{16}$$

where $\gamma_1, \gamma_2, \gamma_{\tilde{W}}, \bar{\gamma}^2_{I_N-W}$ are constants are defined in our main paper ([Gurbuzbalaban et al.(2024)Gurbuzbalaban, Islam, Wang, and Zh

Generalized EXTRA SGLD

Theorem (continues)

Then, for any $K \geq K_0$, the following bound holds:

$$\mathcal{W}_2\left(\mathcal{L}\left(\bar{x}^{(K)}\right), \pi\right) \leq \left(\frac{\bar{\gamma}_{\tilde{W}}^{2K} - \left(1 - \eta\mu\left(1 - \frac{\eta L}{2}\right)\right)^K}{\bar{\gamma}_{\tilde{W}}^2 - 1 + \eta\mu\left(1 - \frac{\eta L}{2}\right)}\right)^{1/2} \cdot \frac{2L\bar{\gamma}_{\tilde{W}}}{\sqrt{N}}\left(\mathbb{E}\left\|x^{(0)}\right\|^2\right)^{1/2} + (1 - \eta\mu)^K \mathcal{W}_2\left(\mathcal{L}(x_0), \pi\right) + \sqrt{\eta}\mathcal{E}_1$$

Generalized EXTRA SGLD

Theorem (continues)

where

$$\mathcal{E}_1 := \left(\frac{\eta}{\mu\left(1-\frac{\eta L}{2}\right)} + \frac{(1+\eta L)^2}{\mu^2\left(1-\frac{\eta L}{2}\right)^2} \right)^{1/2} \cdot \left(\frac{4L^2\left(R_h + R_h'\right)\eta}{N(1-\bar{\gamma}_{\tilde{W}})^2} + \frac{4L^2\sigma^2\eta}{1-\bar{\gamma}_{\tilde{W}}^2} + \frac{8L^2 d}{1-\bar{\gamma}_{\tilde{W}}^2} \right)^{1/2} + \frac{\sigma}{\sqrt{\mu\left(1-\frac{\eta L}{2}\right)N}} + \frac{1.65L}{\mu}\sqrt{dN^{-1}}.$$

Generalized EXTRA SGLD

Theorem (continues)

Moreover,

$$\frac{1}{N}\sum_{i=1}^{N}\mathcal{W}_2\left(\mathcal{L}\left(x_i^{(K)}\right),\pi\right)$$

$$\leq \eta\cdot\frac{D_1}{\sqrt{N}}+\sqrt{\eta}\cdot(D_2+\mathcal{E}_1)+\left(\frac{\bar{\gamma}_{\tilde{W}}^{2K}-\left(1-\eta\mu\left(1-\frac{\eta L}{2}\right)\right)^K}{\bar{\gamma}_{\tilde{W}}^{2}-1+\eta\mu\left(1-\frac{\eta L}{2}\right)}\right)^{1/2}\cdot$$

$$\frac{2L\bar{\gamma}_{\tilde{W}}}{\sqrt{N}}\left(\mathbb{E}\left\|x^{(0)}\right\|^2\right)^{1/2}+(1-\mu\eta)^K\mathcal{W}_2\left(\mathcal{L}\left(x_0\right),\pi\right)+$$

$$\frac{2\left(\bar{\gamma}_{\tilde{W}}\right)^K}{\sqrt{N}}\sqrt{\mathbb{E}\left\|x^{(0)}\right\|^2}$$

Why EXTRA SGLD is better then DE-SGLD

For DE-SGLD, under the assumptions in Theorem 1 in [7], as $\varepsilon \to 0$,

$$\frac{1}{N}\sum_{i=1}^{N} \mathcal{W}_2^{\text{de-sgld}}\left(\mathcal{L}\left(x_i^{(K)}\right), \pi\right) \leq O(\varepsilon), \tag{17}$$

provided that

$$K \geq K^{\text{de-sgld}} = \tilde{O}\left(\frac{L^4 d}{\varepsilon^2 \mu^3}\right) \tag{18}$$

where $\tilde{O}$ hides the logarithmic dependence on ε.

[7] Gürbüzbalaban, Mert, et al. "Decentralized stochastic gradient Langevin dynamics and Hamiltonian monte carlo." Journal of Machine Learning Research 22.239 (2021): 1-69.

Why EXTRA SGLD is better then DE-SGLD

For generalized EXTRA SGLD, under the assumptions in Theorem, as $\varepsilon \to 0$, by taking $h \geq \Omega(\eta\mu)$ and $h < \frac{1}{(L/\mu)^4(L+\|B\|^2)} \wedge \frac{1}{2\gamma_1\gamma_2}$, it holds that

$$\frac{1}{N}\sum_{i=1}^{N} \mathcal{W}_2^{\text{extra-sgld}}\left(\mathcal{L}\left(x_i^{(K)}\right), \pi\right) \leq \mathcal{O}(\varepsilon), \tag{19}$$

provided that

$$K \geq K^{\text{extra-sgld}} = \tilde{\mathcal{O}}\left(\frac{L^2 d}{\epsilon^2 \mu^3}\right), \tag{20}$$

where $\tilde{\mathcal{O}}$ hides the logarithmic dependence on ε and the constants γ_1, γ_2 are provided in the main paper

Experimentally: EXTRA-SGLD vs DE-SGLD

- Synthetic data that we generate by simulating the following model:

$$\delta_i \sim \mathcal{N}(0, \xi^2), \qquad X_i \sim \mathcal{N}(0, I_2), \qquad y_i = \beta^\top X_i + \delta_i, \tag{21}$$

  - white noise δ_i's are i.i.d. scalars with $\xi = 1, \beta \in \mathbb{R}^2$
  - I_2 is the 2×2 identity matrix
- The prior distribution of β follows $\mathcal{N}(0, \lambda I_2)$, and we set $\lambda = 10$ for this set of experiments.
- The posterior distribution can be derived from the following model

$$\pi(\beta) \sim \mathcal{N}(m, V) \quad m := \left(\Sigma^{-1} + \frac{X^\top X}{\xi^2}\right)^{-1} \left(\frac{X^\top y}{\xi^2}\right),$$

$$V := \left(\frac{X^\top X}{\xi^2} + \Sigma^{-1}\right)^{-1},$$

Experimentally: EXTRA-SGLD vs DE-SGLD

- where $\Sigma = \lambda I_2$ is the covariance matrix of the prior of β
- $X = \left[X_1^\top, X_2^\top, \cdots\right]^\top$ and $Y = [y_1, y_2, \cdots]^\top$ are the input and output matrices, respectively.
- For this experiment, we simulate 5000 data points using the model (21) and then we distribute these data points randomly among the $N = 20$ agents.
- All agents have an equal amount of data exclusively, and share only the parameter estimates.

Experimentally: EXTRA-SGLD vs DE-SGLD

The posterior distribution $\pi(\beta) \propto e^{-f(\beta)}$ where $f(\beta) = \sum_{i=1}^{N} f_i(\beta)$ with

$$f_i(\beta) := -\sum_{j=1}^{n_i} \log p\left(y_j^i | \beta, X_j^i\right) - \frac{1}{N} \log p(\beta) = \sum_{j=1}^{n_i} \left(y_j^i - \beta^\top X_j^i\right)^2 + \frac{1}{2\lambda N} \|\beta\|^2,$$

where

$$p\left(y_j^i | \beta, X_j^i\right) = \frac{1}{\sqrt{2\pi\xi^2}} e^{-\frac{1}{2\xi^2}(y_j^i - \beta^\top X_j^i)^2}, \quad p(\beta) \propto e^{-\frac{1}{2\lambda}\|\beta\|^2},$$

and each agent i has an equal number of $n_i = 50$ data points $\{(X_j^i, y_j^i)\}_{j=1}^{n_i}$.

Experimentally: EXTRA-SGLD vs DE-SGLD

- We restrict the experiments with a deterministic gradient, i.e. $\sigma = 0$ and a fixed stepsize $\eta = 0.009$.
- The doubly stochastic mixing matrix $\widetilde{W} = hI_N - (1-h)W$ is calculated for different values of the parameter h.
- We consider 5 linearly spaced h values with the minimum being 0.001 and the maximum being 0.5 and we tune up the parameter h to the network. For the fully connected network $h = 0.50$, circular network $h = 0.38$, star network $h = 0.13$, and for the disconnected network $h = 0.38$.
- For each agent i, the iterations $\beta_i^{(k)} \sim \mathcal{N}\left(m_i^{(k)}, \Sigma_i^{(k)}\right)$ and we compute the Wasserstein-2 distance based on the formula given by Givens[8]

[8] Givens, Clark R., and Rae Michael Shortt. "A class of Wasserstein metrics for probability distributions." Michigan Mathematical Journal 31.2 (1984): 231-240.

Experimentally: EXTRA-SGLD vs DE-SGLD

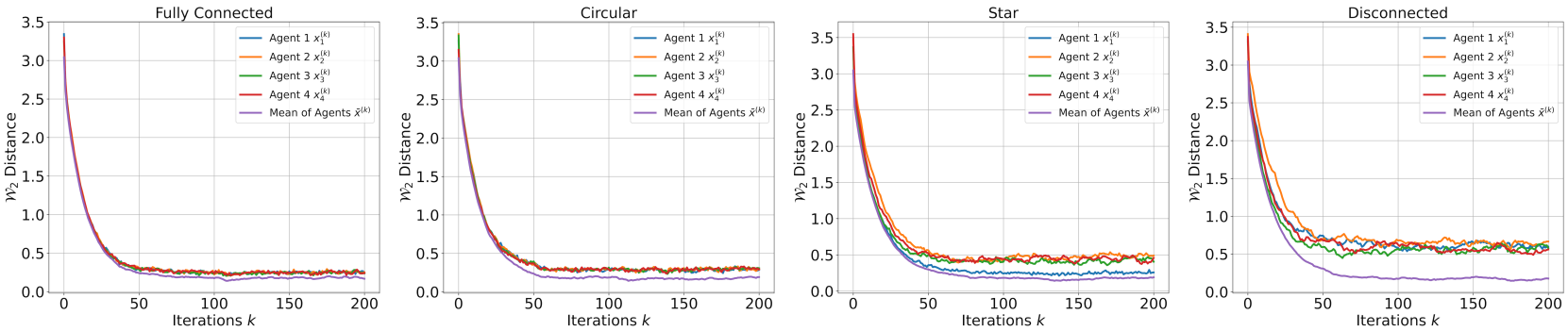


Figure 3: Performance of the EXTRA SGLD for Bayesian linear regression on four different network structures. Out of 20 agents, we report only the first 4 agents and the mean of the nodes $\bar{\beta}^{(k)} = \frac{1}{N}\sum_{i=1}^{N} \beta_i^{(k)}$.

Experimentally: EXTRA-SGLD vs DE-SGLD

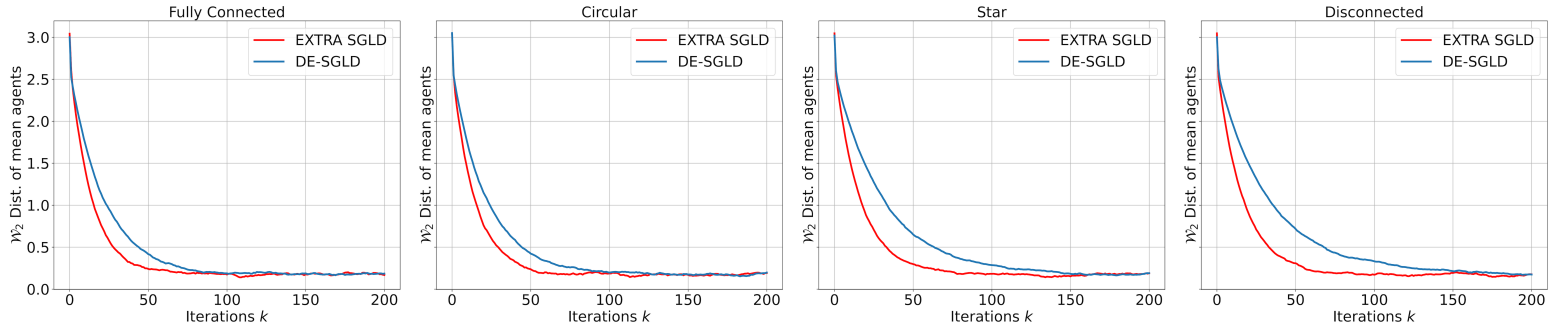


Figure 4: Comparative performance of the DE-SGLD and EXTRA SGLD for Bayesian linear regression on four different network structures in terms of the $\mathcal{W}_2$ distance of mean agents

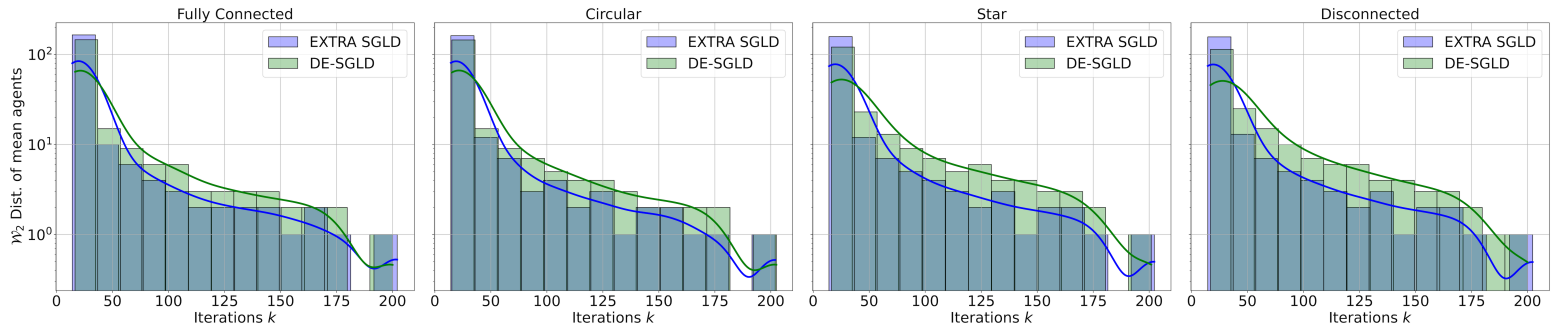


Figure 5: Histogram of the comparative performances of the DE-SGLD and EXTRA SGLD for Bayesian linear regression on four different network structures

Experimentally: EXTRA-SGLD vs DE-SGLD

- We have a dataset $Z = \{z_j\}_{j=1}^n$ where $z_j = (X_j, y_j), X_j \in \mathbb{R}^d$ are the features and $y_j \in \{0, 1\}$ are the labels with the assumption that X_j are independent.
- The probability distribution of y_j given X_j and regression coefficients $\beta \in \mathbb{R}^d$ is given by $\mathbb{P}(y_j = 1|X_j, \beta) = \frac{1}{1+e^{-\beta^\top X_j}}$.
- The prior distribution $p(\beta) \sim \mathcal{N}(0, \lambda I_d)$ for some $\lambda > 0$, where I_d is the $d \times d$ identity matrix.

Experimentally: EXTRA-SGLD vs DE-SGLD

- In a distributed network system, if each agent i contains a subset Z_i of data, then the goal of the Bayesian logistic regression is to sample from $\pi(\beta) \propto e^{-f(\beta)}$ with $f(\beta) = \sum_{i=1}^{N} f_i(\beta)$ where

$$\begin{aligned} f_i(\beta) &:= -\sum_{j=1}^{n_i} \log p\left(y_j^i = 1 | X_j^i, \beta\right) - \frac{1}{N} \log p(\beta) \\ &= \sum_{j=1}^{n_i} \log\left(1 + e^{-\beta^\top X_j}\right) + \frac{1}{2N\lambda}\|\beta\|^2 \end{aligned} \tag{22}$$

  is strongly convex and smooth.
- Data: Wisconsin Breast Cancer Data

Experimentally: EXTRA-SGLD vs DE-SGLD

- Data: Wisconsin Breast Cancer Data
- The data set contains 569 instances with 30 features which are computed from a digitized image of a fine needle aspirate (FNA) of a breast mass.
- For this experiment, we take:
  - $\eta = 0.005$
  - batch size 32
  - $N = 6$
- We take $h = 0.278, 0.389, 0.167$, and 0.278 for fully connected network, circular network, star network, and disconnected network, respectively.

Experimentally: EXTRA-SGLD vs DE-SGLD

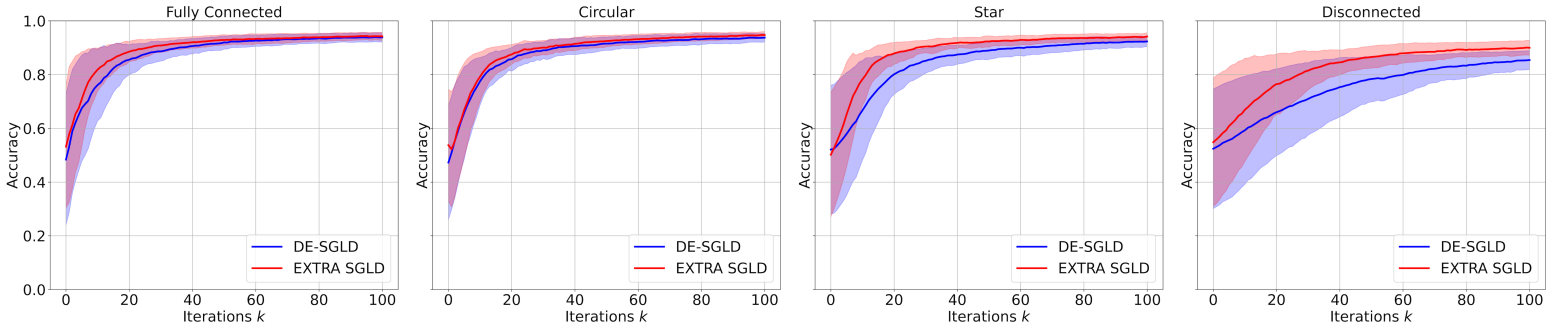


Figure 6: Comparative accuracy distribution of the DE-SGLD and EXTRA SGLD method across different network structures on Breast Cancer data set. The plots are from a randomly selected node.

Experimentally: EXTRA-SGLD vs DE-SGLD

	FCN	CN	SN	FDN (Cons.)	FDN (Ens.)
DE-SGLD	98.80%	98.79%	99.08%	11.79%	99.08%
EXTRA SGLD	98.91%	98.92%	98.96%	15.31%	99.18%

Table 1: MNIST classification accuracies across different network topologies

Bibliography


Mert Gurbuzbalaban, Mohammad Rafiqul Islam, Xiaoyu Wang, and Lingjiong Zhu.
Generalized extra stochastic gradient langevin dynamics.
arXiv preprint arXiv:2412.01993, 2024.